Practice Course in Plasma

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Thursday 9:10-12:00

Material: http://capst.ncku.edu.tw/PGS/index.php/teaching/

Lecture 8

2021/4/22 updated

A magnetic field of 0.0876 T is needed for ECH

$$\omega_{ce} = \frac{eB}{m_e c} \equiv \omega = 2\pi \times 2.45\ \mathrm{GHz}$$

$$B = \frac{2\pi \times 2.45 \times 10^9 m_e c}{e} = 876\ \mathrm{G} = 0.0876\ \mathrm{T}$$

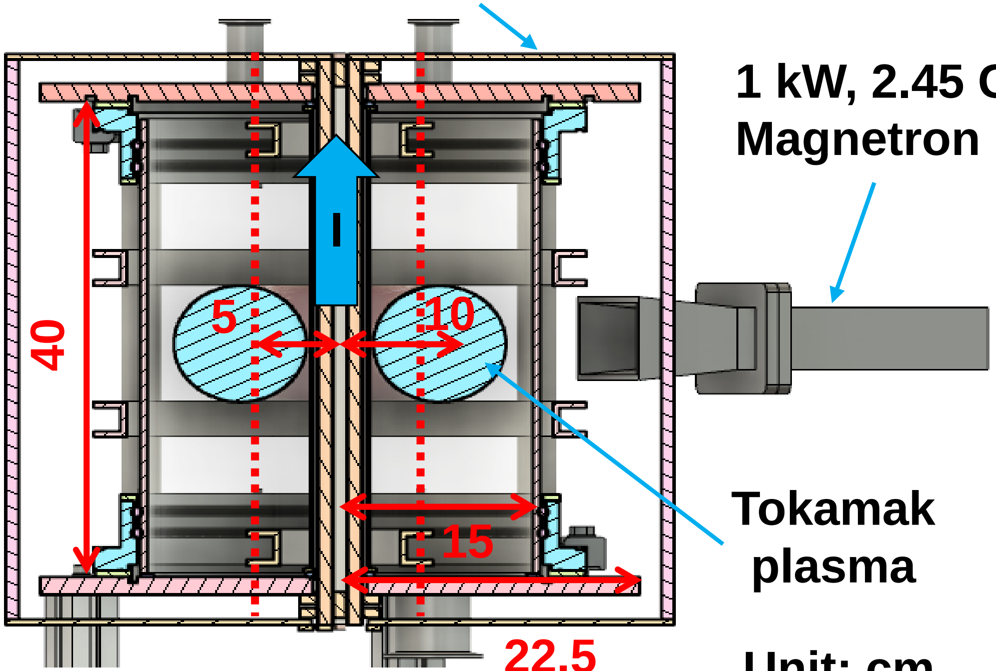
$$B = \frac{\mu_0 I}{2\pi r}$$

$$I = \frac{2\pi r B}{\mu_0} = \frac{2\pi r B}{4\pi \times 10^{-7}}$$

$$= 5 B_{(\mathrm{T})} r_{(\mathrm{m})} (\mathrm{MA})$$

$$= 22\ \mathrm{kA}\ @\ 5\ \mathrm{cm}$$

Toroidal field coil



1 kW, 2.45 GHz Magnetron

Tokamak plasma

Unit: cm

- A pulsed-power system will be used to generate the current with a pulse width of 1 ms.

The 0.1-T magnetic field is sufficient to confine 10-eV Ar ion

$$v = r\omega \qquad \frac{1}{2}mv^2 = kT \qquad \omega = \frac{eB}{m} \qquad r = \frac{\sqrt{2mkT}}{eB}$$

- **Larmor radius in mm @ B=0.1 T:**

T (eV)	H (1g/mole)	D (2g/mole)	T (3g/mole)	He (4g/mole)	Ar (40g/mole)
1	1.4	2.0	2.5	2.9	9.1
10	4.6	6.5	7.9	9.1	28.9
100	14.4	20.4	25.0	28.9	91.3
1000	45.6	64.5	79.1	91.3	288.7

- **The Larmor radius of 1-keV electron @ B=0.1 T is 1.1 mm. Electrons are confined in our system.**

- **Ar will be used.**

The magnetic field energy of the toroidal field is ~100 J

$$B = \frac{\mu_0 I}{2\pi r}$$

$$E = \int \frac{B^2}{2\mu_0} \mathrm{dv} = \int_{r_{\min}}^{r_{\max}} \frac{B^2}{2\mu_0} 2\pi r L \, \mathrm{d}r = \frac{2\pi L}{2\mu_0} \frac{{\mu_0}^2 I^2}{(2\pi)^2} \int_{r_{\min}}^{r_{\max}} \frac{1}{r^2} r \, \mathrm{d}r = \frac{\mu_0 L I^2}{4\pi} \int_{r_{\min}}^{r_{\max}} \frac{1}{r} \mathrm{d}r$$

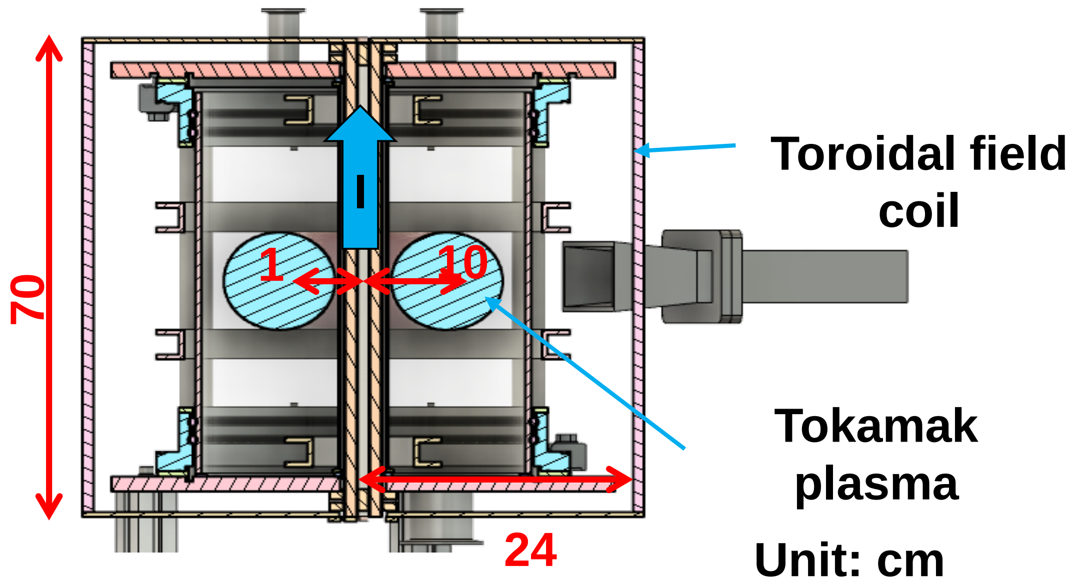
$$= \frac{\mu_0 L I^2}{4\pi} \ln\left(\frac{r_{\max}}{r_{\min}}\right) = 10^{-7} \times 0.7 \times \left(20 \times 10^3\right)^2 \ln\left(\frac{24}{1}\right)$$

$$= 89 \text{ J}$$

$$E = \frac{1}{2} L I^2 \qquad L = \frac{2E}{I^2}$$

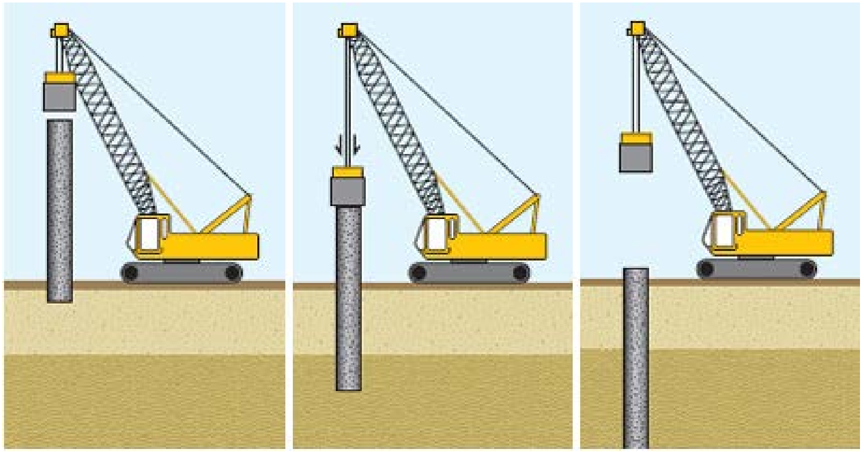
#/ of Turns	*I* (kA)	*L* (μH)
1	20	0.45
4	5	7.1
8	2.5	28.5

- Measured *L*=15 μH



A pulsed-power system is capable of providing a high-power output

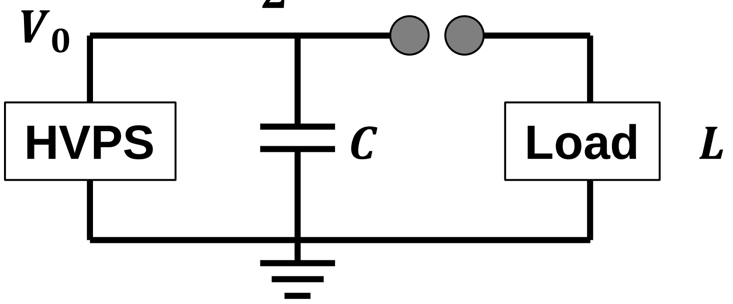
- **Driven piles**



PLACEMENT OF PILE INSTALLATION OF PILE REPETITION OF PROCESS

- **Capacitive-storage pulsed-power system**

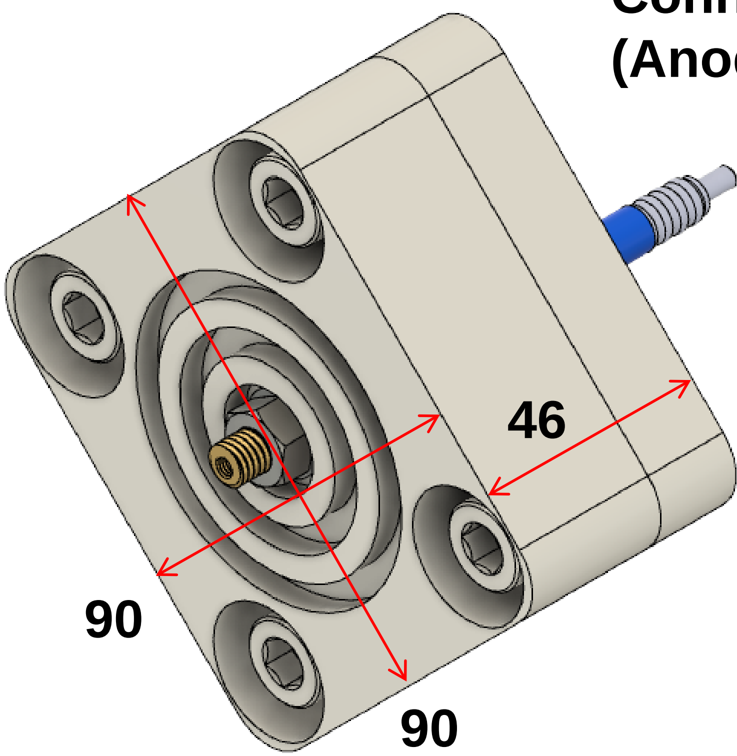
$$E = \frac{1}{2} C V_0^2$$

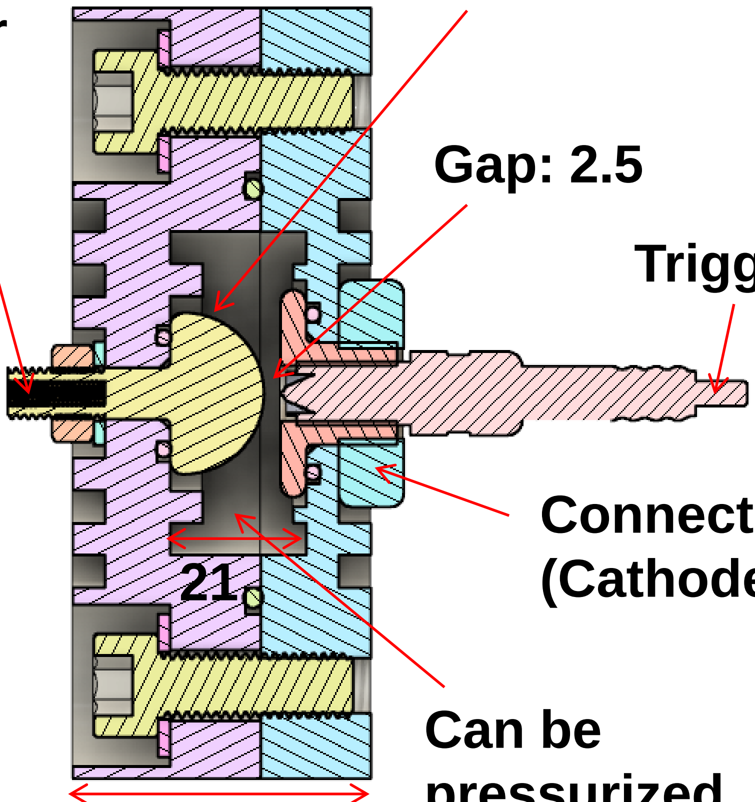


Low power IN **High power OUT**

http://www.saudifoundations.com/driven.html

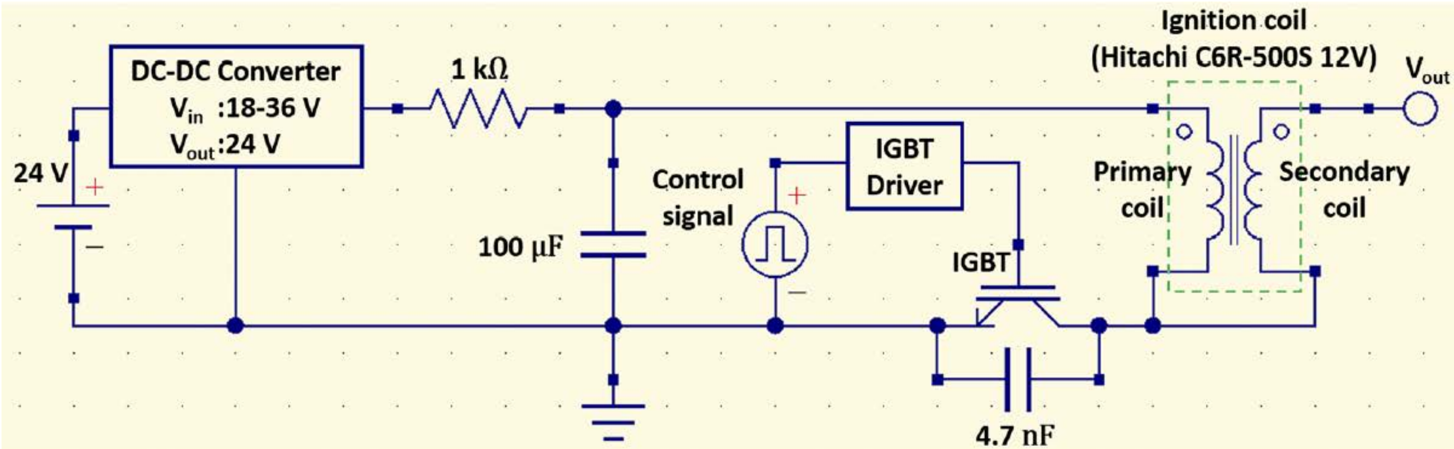
Trigatron will be used as the controlled-spark gap switch

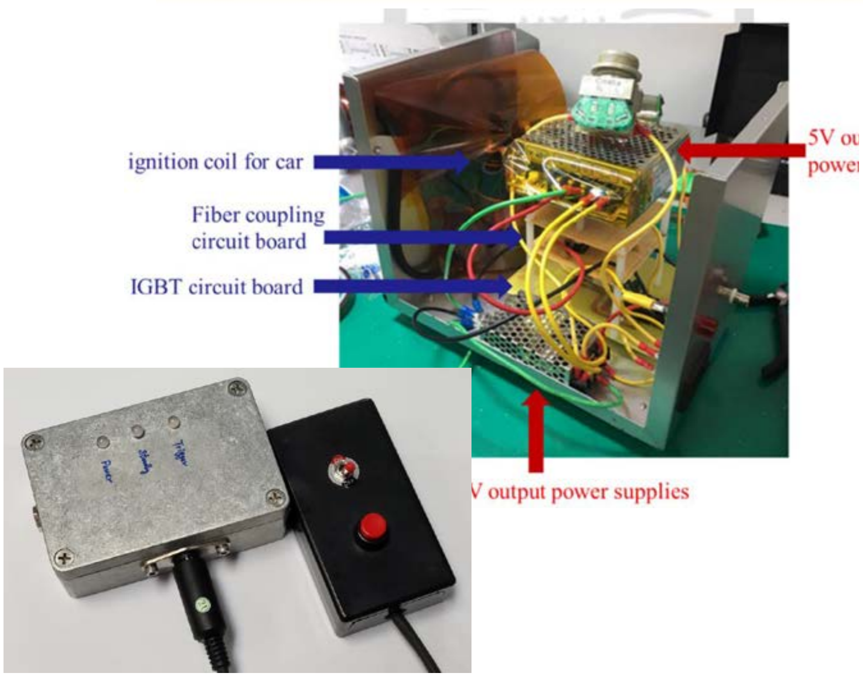


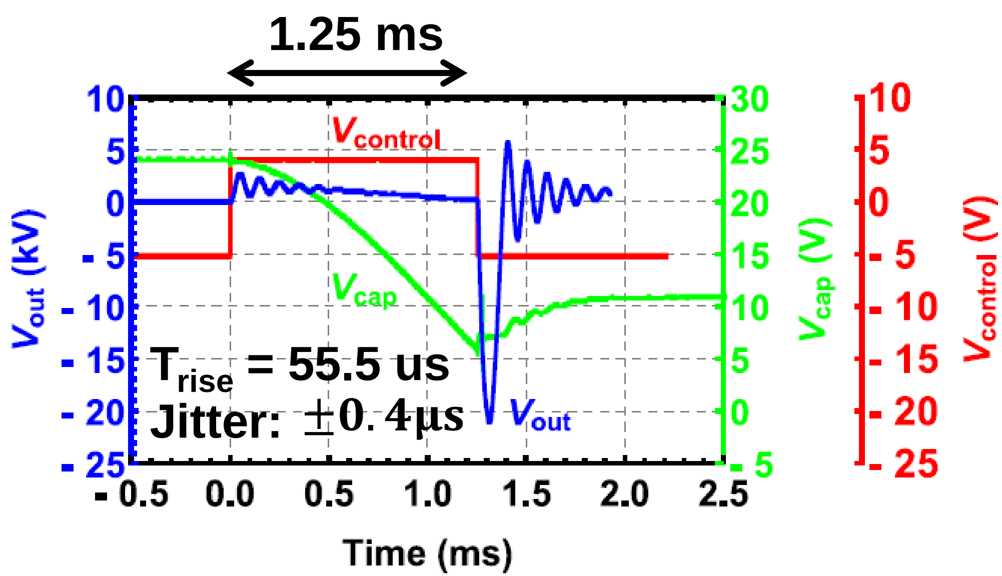


Unit: mm

A slow trigger pulse generator was built using a ignition coil for cars

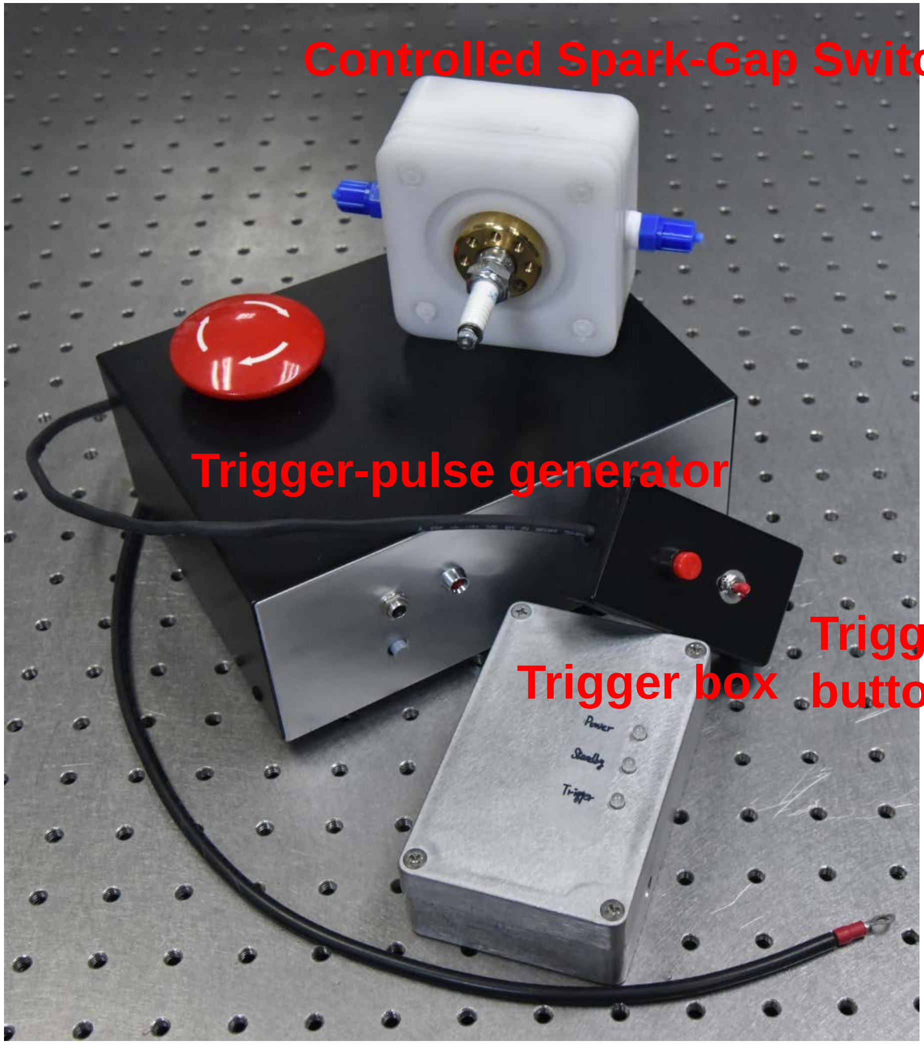






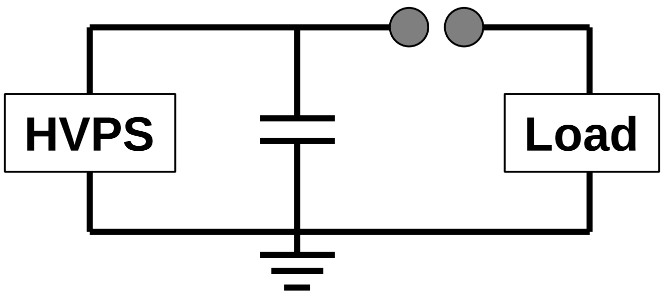
P.-Y. Chang etc. Rev. Sci. Instrum. 91, 114703 (2020)

The controlled spark-gap switch

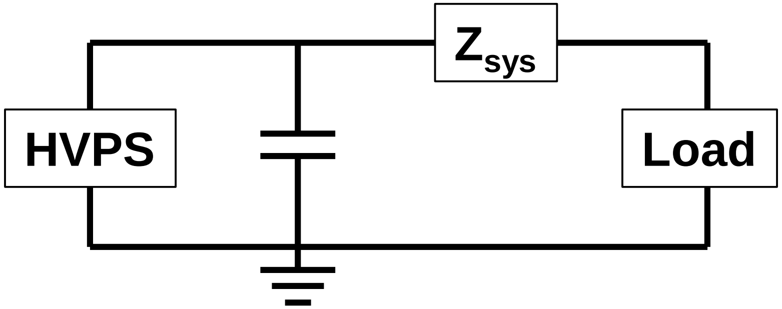


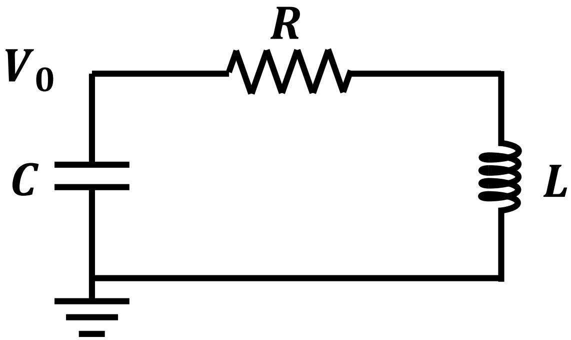
A simple pulsed-power system is a RLC circuit

- **Before discharge**

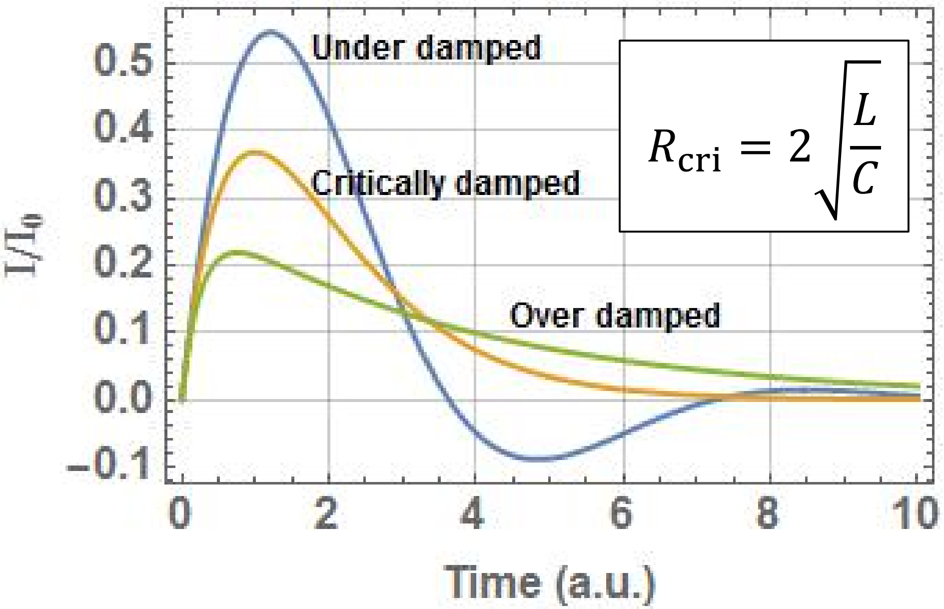


- **After discharge**

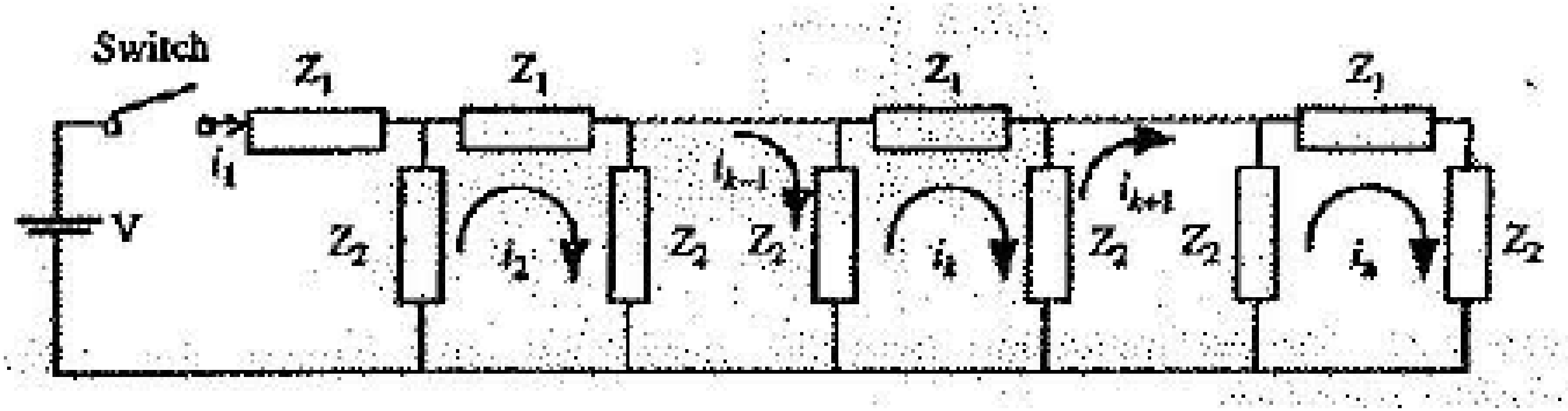




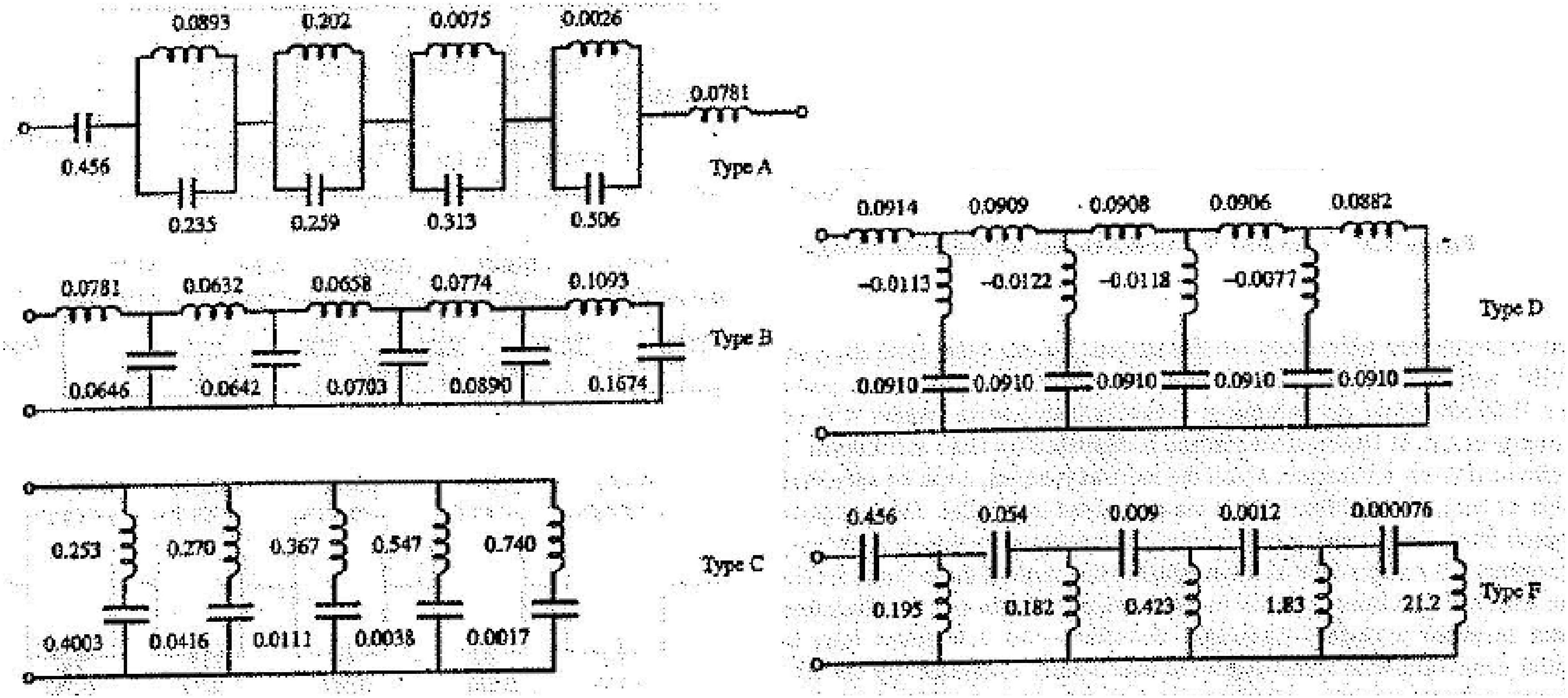
- **How can we generate a square current pulse?**



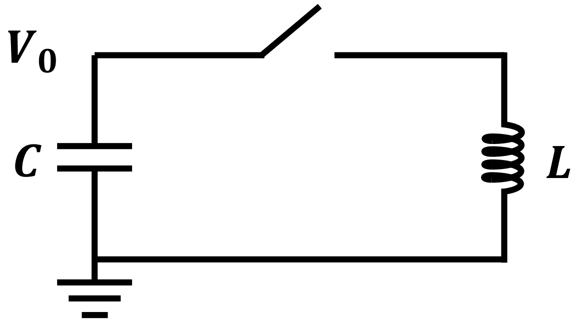
Pulse-forming network (PFN)



Different type of PFN



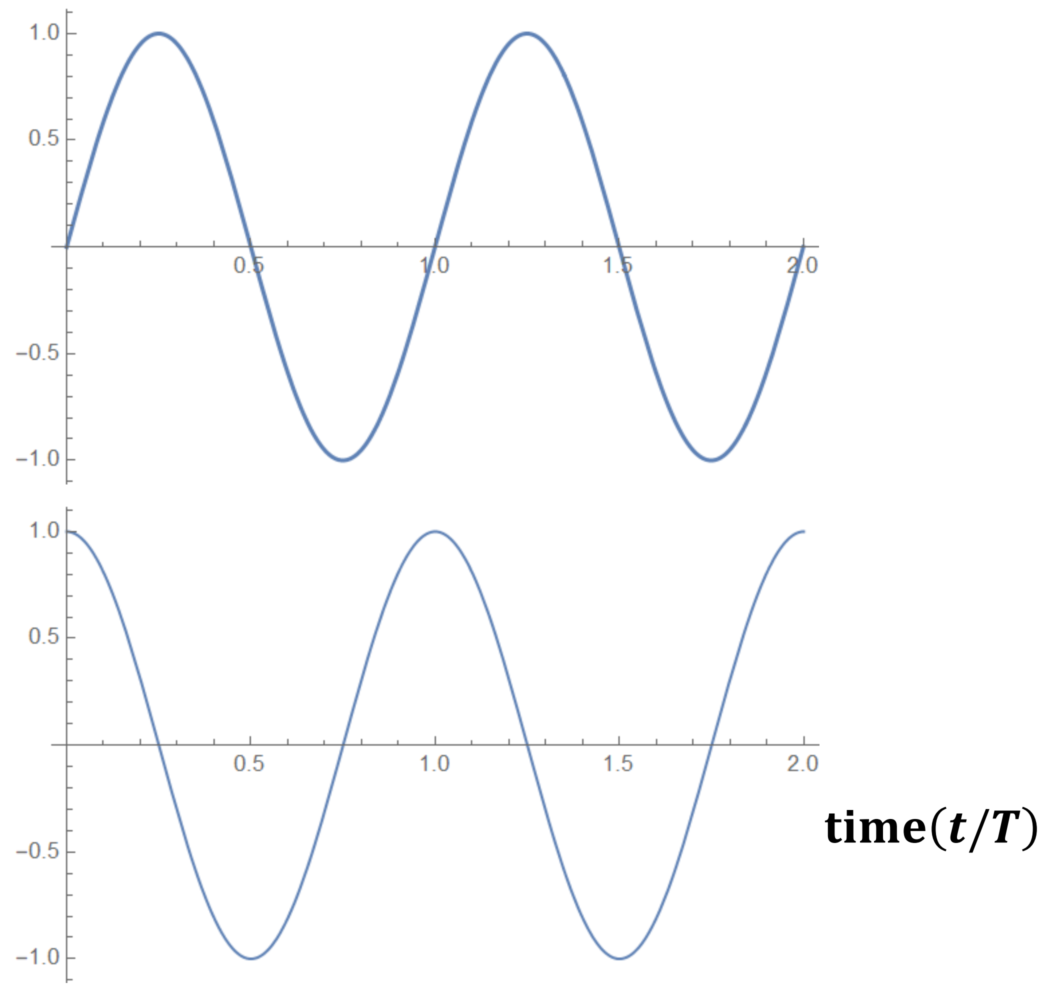
The current output of a LC circuit is a basis of Fourier series



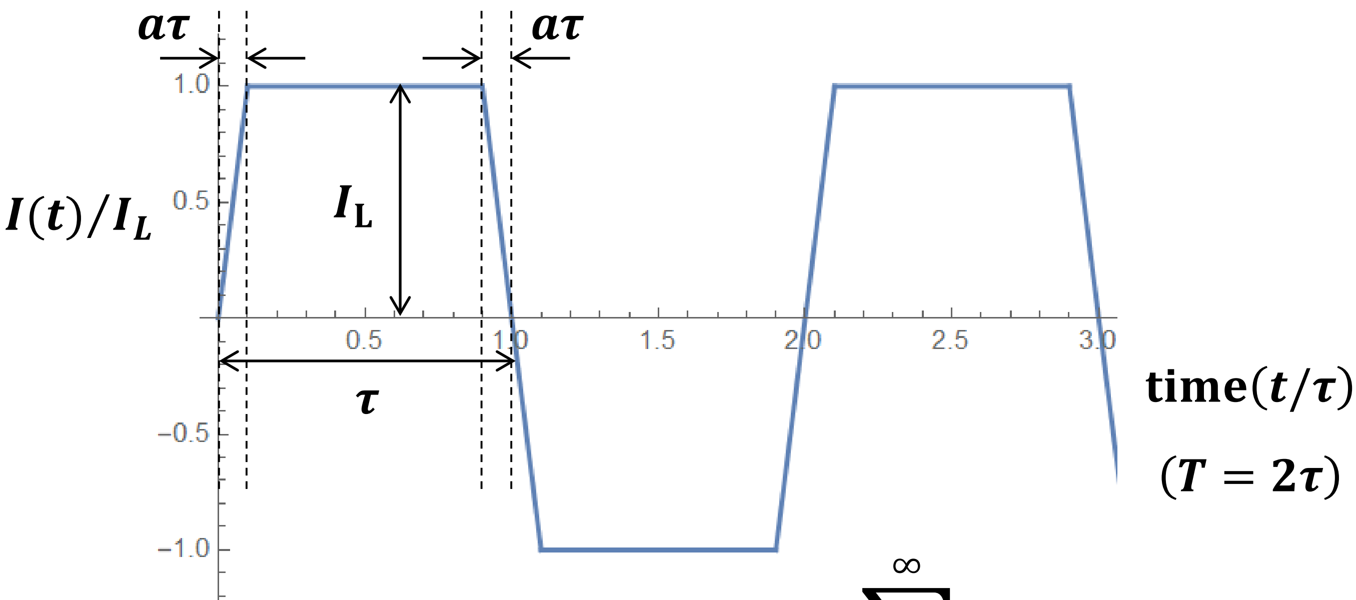
$$I(t) = V_0\sqrt{\frac{C}{L}}\sin\left(\frac{t}{\sqrt{LC}}\right)$$

$$V(t) = V_0\cos\left(\frac{t}{\sqrt{LC}}\right)$$

$$Z = \sqrt{\frac{L}{C}} \qquad \omega = \frac{1}{\sqrt{LC}}$$



A trapezoidal wave can be expressed by Fourier series (Guillemin's method)



$$\frac{i(t)}{I_L} = \frac{t}{a\tau} \quad , 0 \le t \le a\tau$$

$$\frac{i(t)}{I_L} = 1 \quad , a\tau \le t \le \tau - a\tau$$

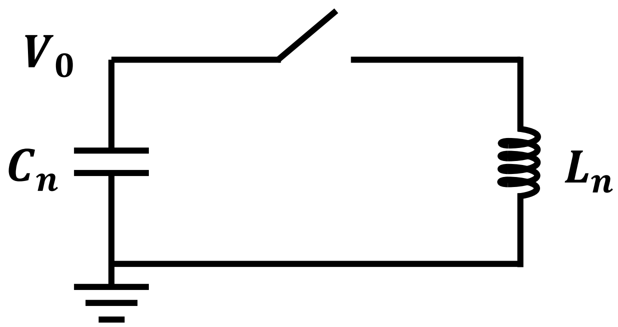
$$\frac{i(t)}{I_L} = \frac{\tau - t}{a\tau} \quad , \tau - a\tau \le t \le \tau$$

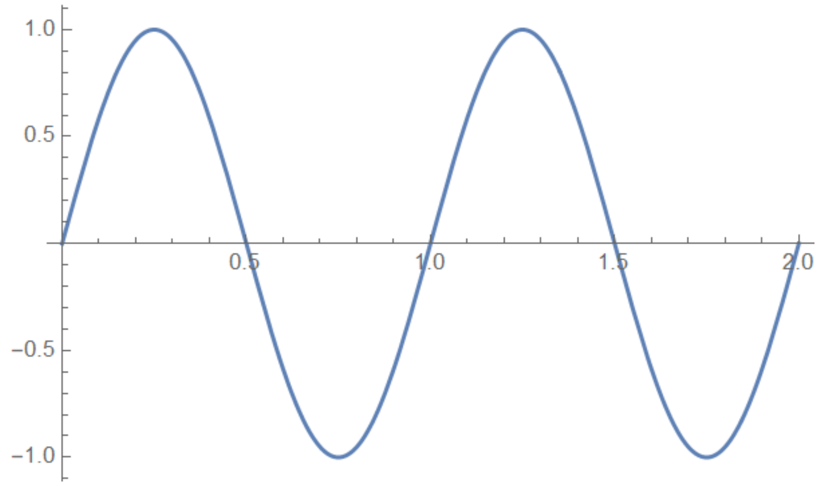
$$i(t) = I_L \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi t}{\tau}\right)$$

where $b_n = \frac{2}{\tau} \int_0^{\tau} \frac{i(t)}{I_L} \sin\left(\frac{n\pi t}{\tau}\right) \mathrm{d}t$

$$b_n = \frac{4}{n\pi} \frac{\sin(n\pi a)}{n\pi a}, \text{where } n = 1, 3, 5 \ldots$$

The required inductance and capacitance are obtained by comparing LC output with the Fourier series





$$I_n(t) = V_0\sqrt{\frac{C_n}{L_n}}\sin\left(\frac{t}{\sqrt{L_nC_n}}\right)$$

$$L_n = \frac{Z_n\tau}{n\pi b_n} = \frac{V}{I_L}\frac{\tau}{n\pi b_n}$$

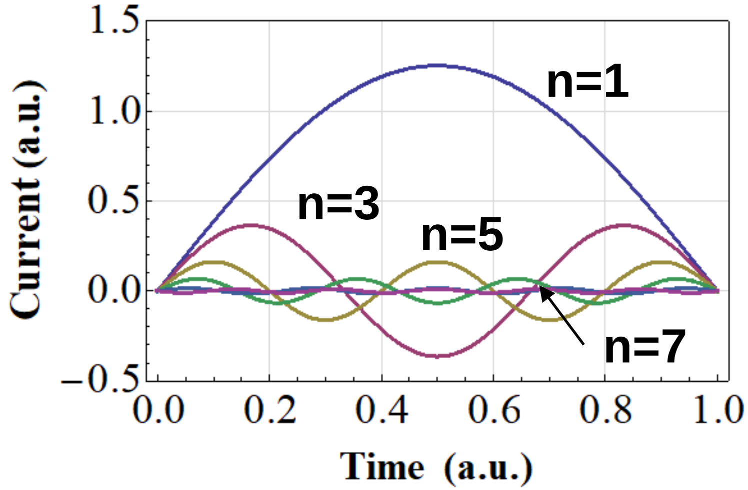
$$i(t) = I_L\sum_{n=1}^{\infty} b_n\sin\left(\frac{n\pi t}{\tau}\right)$$

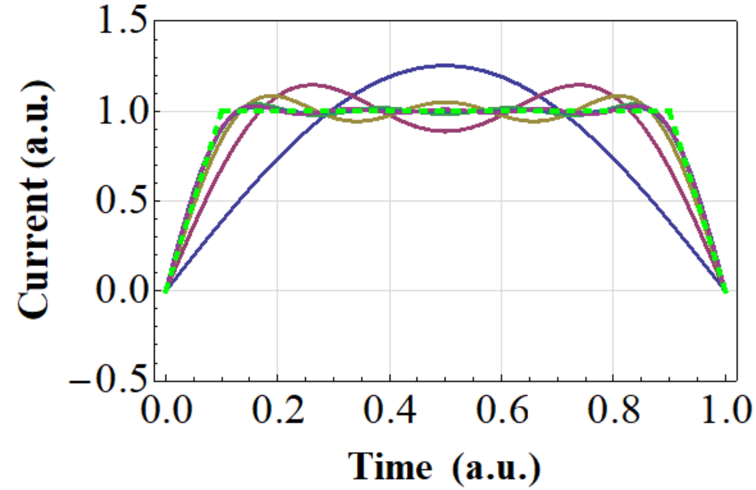
$$C_n = \frac{\tau b_n}{n\pi Z_n} = \frac{I_L}{V}\frac{\tau b_n}{n\pi}$$

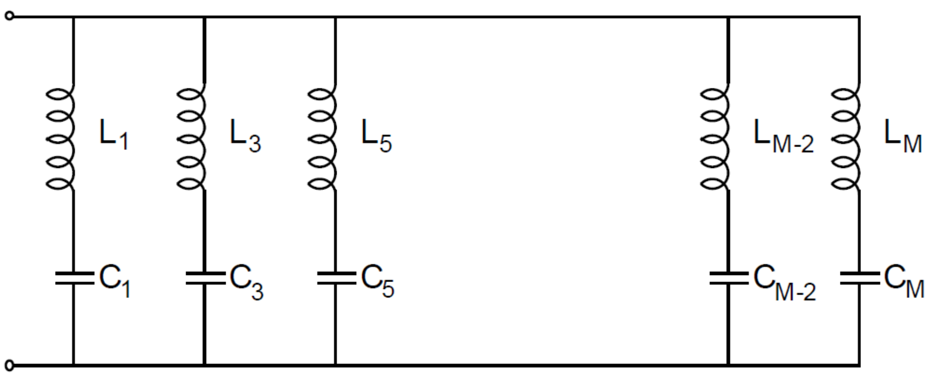
$$Z_n = \frac{V}{I_L}$$

$$b_n = \frac{4}{n\pi}\frac{\sin(n\pi a)}{n\pi a}, \text{where } n = 1, 3, 5\ldots$$

A trapezoidal current output can be generated using Guillemin's pulse-forming networks





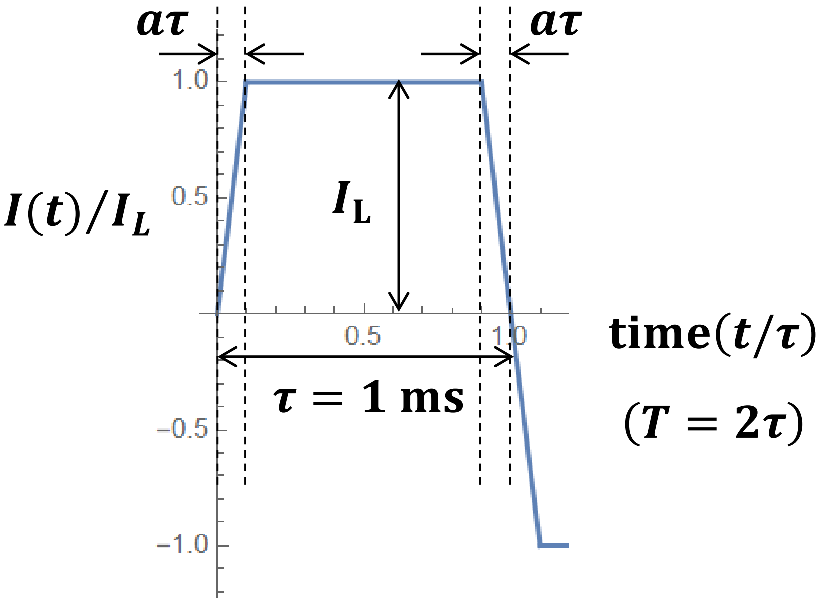


$$I(t) = I_L \Sigma b_n \mathrm{Sin}\left(\frac{n\pi t}{\tau}\right)$$

$$b_n = \frac{4}{n\pi} \frac{\mathrm{Sin}(n\pi a)}{n\pi a}$$

$$L_n = \frac{Z\tau}{n\pi b_n} \quad C_n = \frac{\tau b_n}{n\pi Z} \quad Z = \frac{V}{I_L}$$

Fourier components of τ=1 ms, a=0.1



$$i(t) = I_L \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi t}{\tau}\right)$$

$$b_n = \frac{4}{n\pi}\frac{\sin(n\pi a)}{n\pi a}, \text{where } n = 1, 3, 5 \ldots$$

n	#I
b1	1.2524
b3	0.3643
b5	0.1621
b7	0.069
b9	0.0155

Coils with 8 turns and a PFN charged to 1 kV will be used

I (kA)	V (kV)		1	2	3	4	5	E (kJ)	% to 100 J
2.5	1	L(uH)	101.7	116.5	157.1	271.8	915.0	0.6	17.7 %
		C(uF)	996.6	96.6	25.8	7.6	1.4		
2.5	1	L(uH)	101.7	116.5	157.1	271.8	915.0	0.6	17.7 %
		C(uF)	990	100	25	10	1		
2.5	1	L(uH)	86.4	86.4	135	264.6	912.6	0.6	17.7 %
		C(uF)	990	100	25	7.5	1.5		
2.5	1	L(uH)	86.4	126	131	253	907	0.6	17.7 %
		C(uF)	990	100	25	7.5	1.5		

Reality

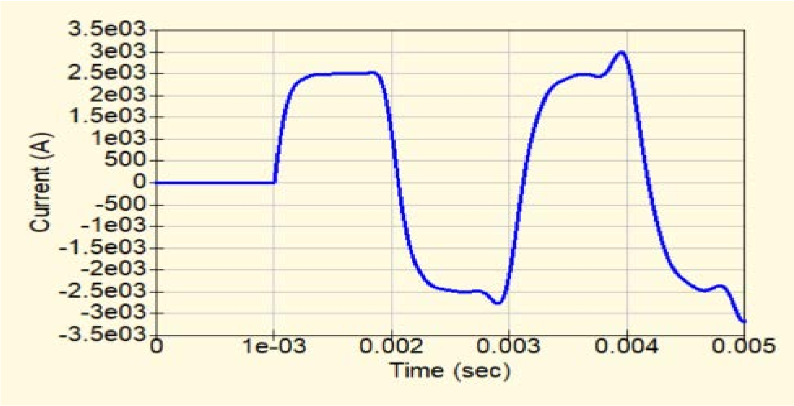
$$\mu_r \approx 5000$$

$$A_L = 5400 \pm 25\% \text{ nH}$$

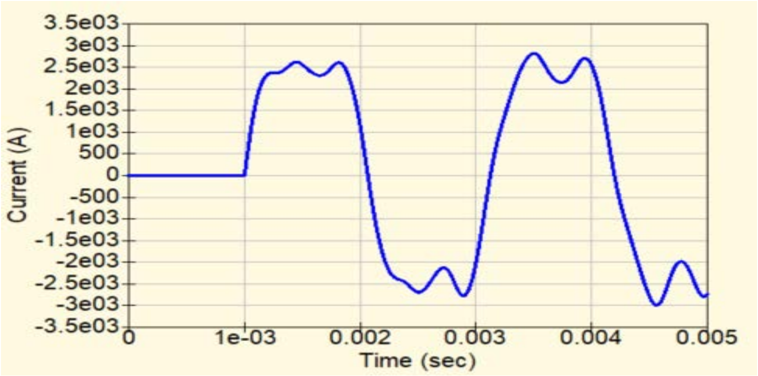
$$L = A_L N^2$$

A square pulse with a flat top of 2.5 kA can be generated

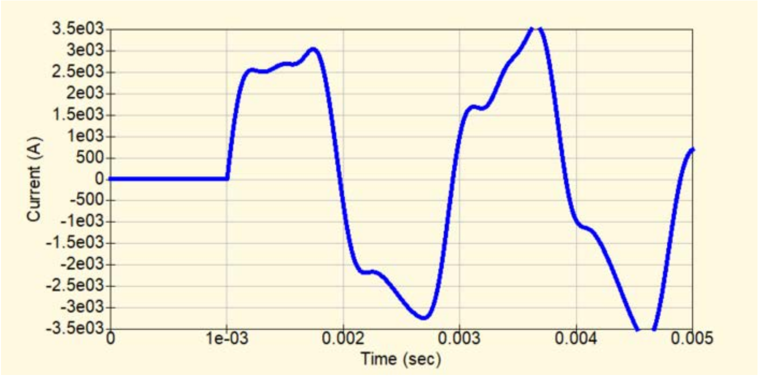
- Case 1: (L-theory, C-theory)



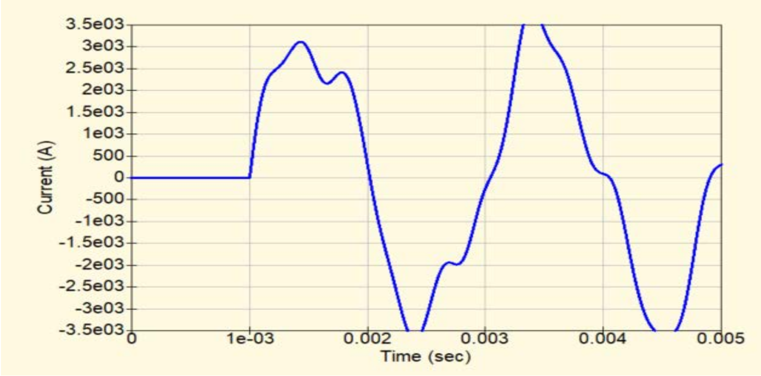
- Case 2: (L-theory, C-real)



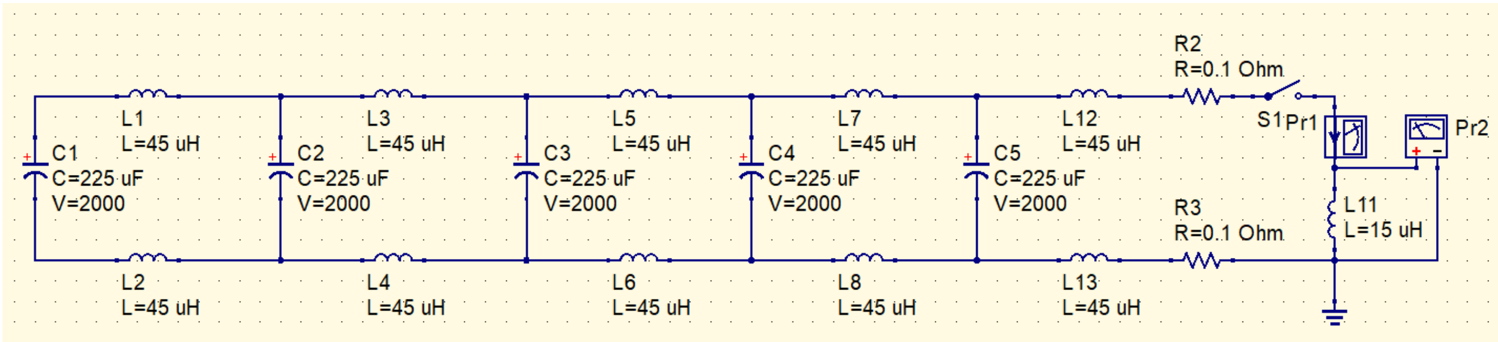
- Case 3: (L-Half theory, C-real)



- Case 4: (L-real, C-real)



A simple PFN with constant C and L in all stages can also be used



$$C \equiv \bar{C} = \frac{1}{N}\sum_{n=1}^{N} C_n = 225\mu\text{F}$$

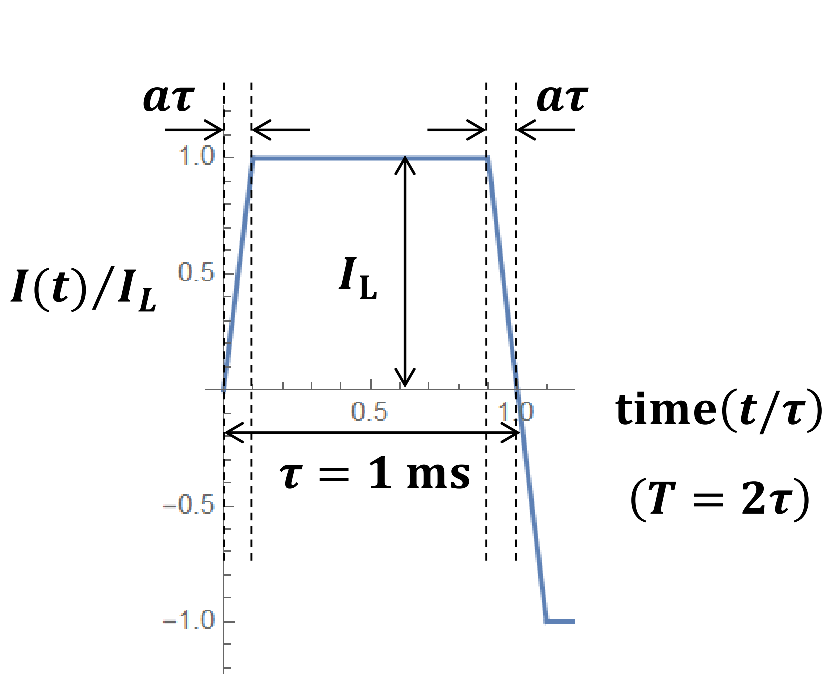
$$L_n = 2nL + L_L \approx 2nL$$

$$\omega_n = \frac{1}{\sqrt{L_n C}} \approx \frac{1}{\sqrt{2nLC}}$$

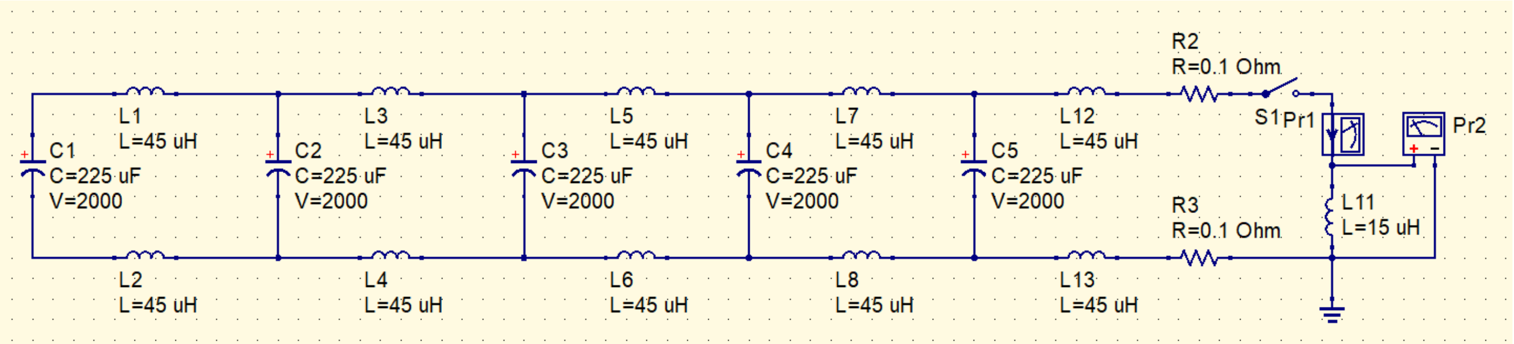
- **For 5 stages:**

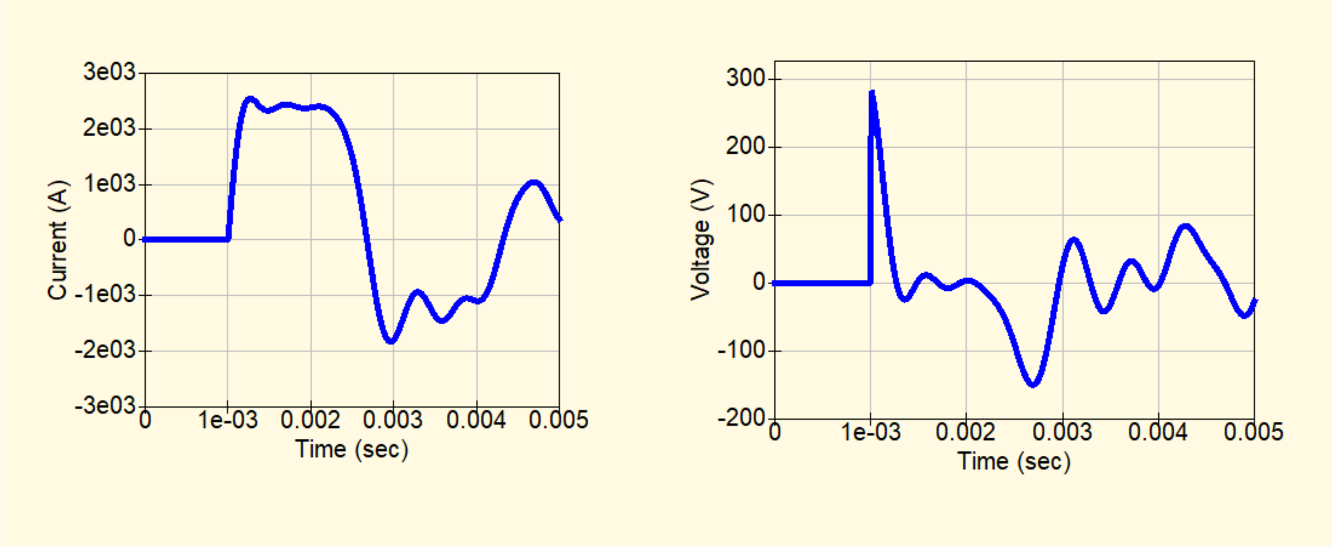
$$\omega_5 = \frac{2\pi}{T} = \frac{\pi}{\tau} = \frac{\pi}{1\text{ms}}$$

$$L = 45\mu\text{H}$$



The energy coupling efficiency is lower using the simple PFN

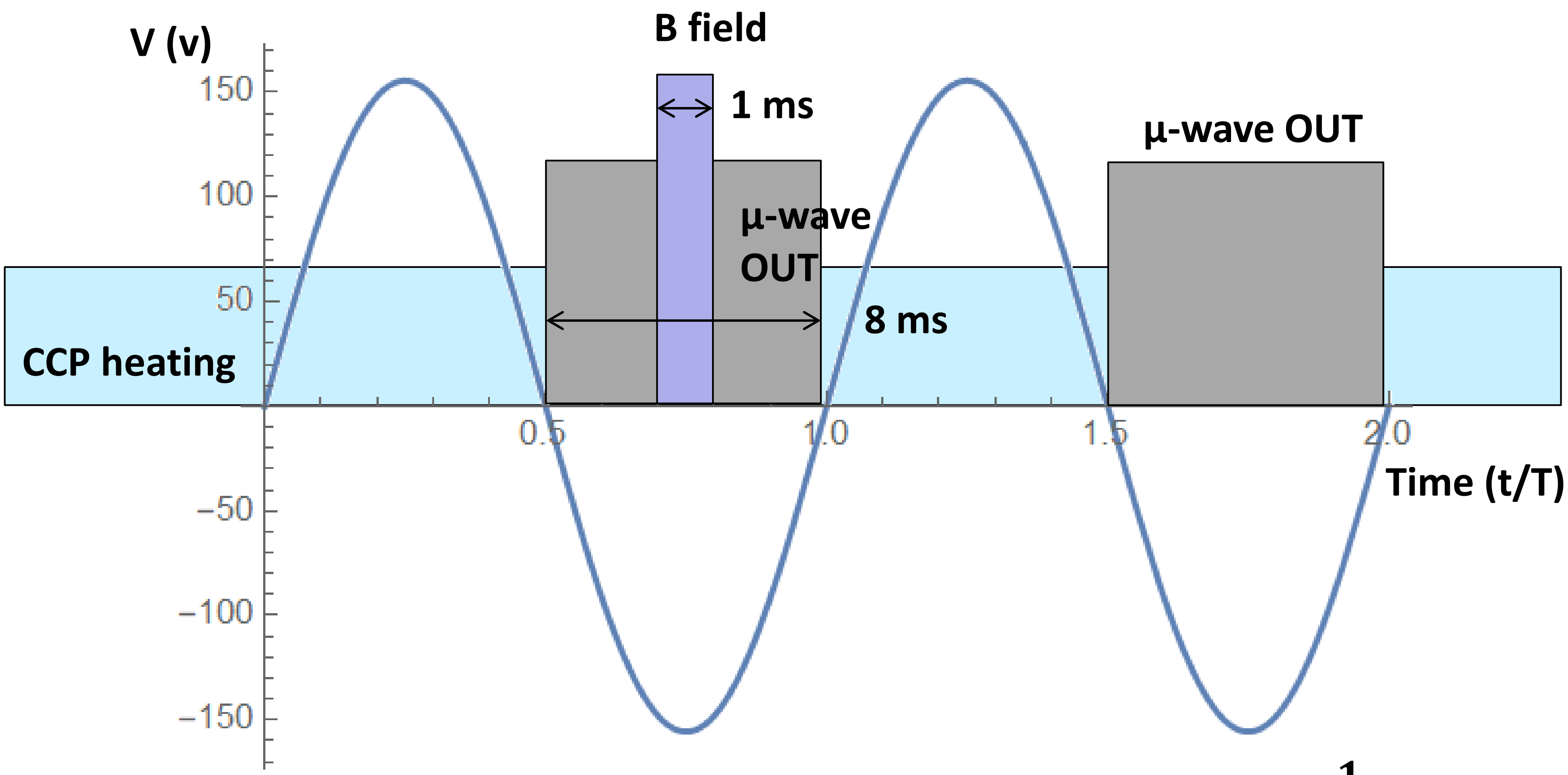




$$E = \frac{1}{2} CV^2 = 2.25\text{kJ}$$

- **Only 4.4 % of the energy is transferred to magnetic energy.**

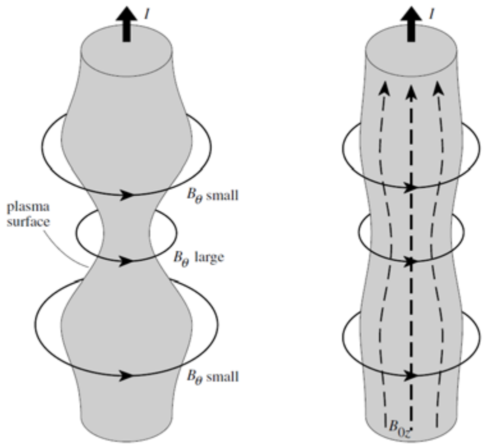
Time sequence



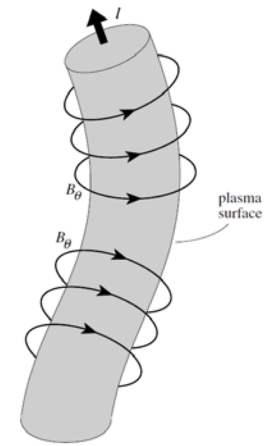
$$T = \frac{1}{60} = 16.7\text{ms}$$

Instabilities occur in a cylindrical plasma column

- **Sausage instability:**



- **Kink instability:**



- **Safety factor *q*:**

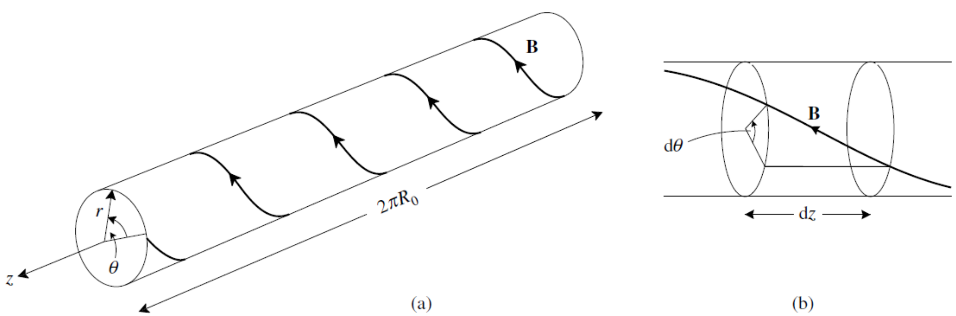
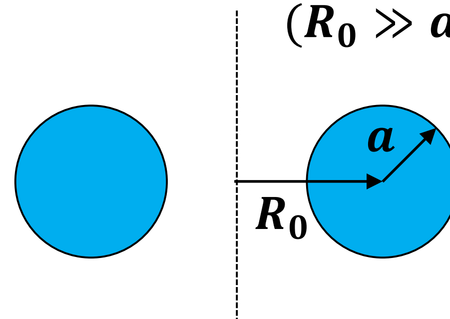


Fig. 4.6. Screw pinch geometry.

$$q(r) = \frac{rB_r(r)}{R_0B_\theta(r)} \approx \frac{rB_t}{R_0B_p}$$

$$(R_0 \gg a)$$



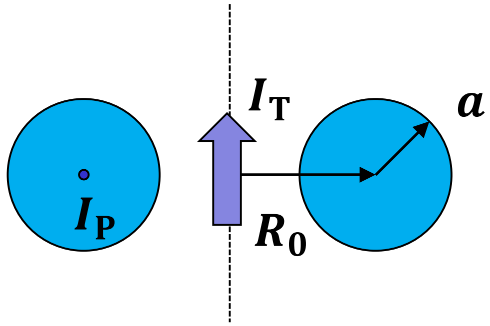
A plasma current of ~ 2 kA is needed

$$q(r) \approx \frac{rB_t}{R_0 B_p} \approx \frac{\mathrm{a}\, B_t}{R_0 B_p}$$

$$B_T = \frac{\mu_0 I_T}{2\pi R_0} \qquad B_P = \frac{\mu_0 I_p}{2\pi a}$$

$$I_p \sim \frac{1}{q}\left(\frac{a}{R_0}\right)^2 I_T = \frac{1}{3}\left(\frac{5}{10}\right)^2 20\mathrm{kA} \sim 2\mathrm{kA}$$

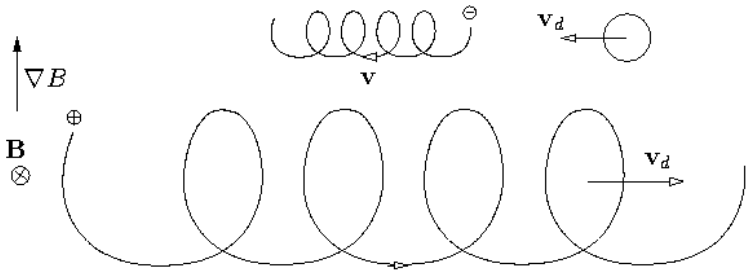
$$B_p \sim 40\ \mathrm{G}$$



$R_0 \sim 10$ cm
a ~ 5 cm

Plasma current will be generated by the Grad-B drift and the Curvature drift current

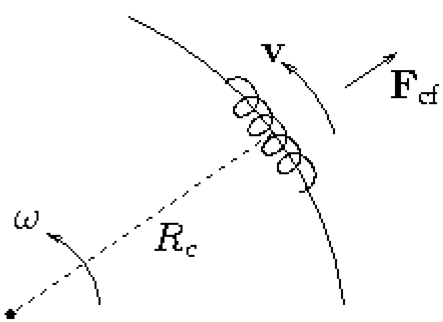
- **Grad-B drift**

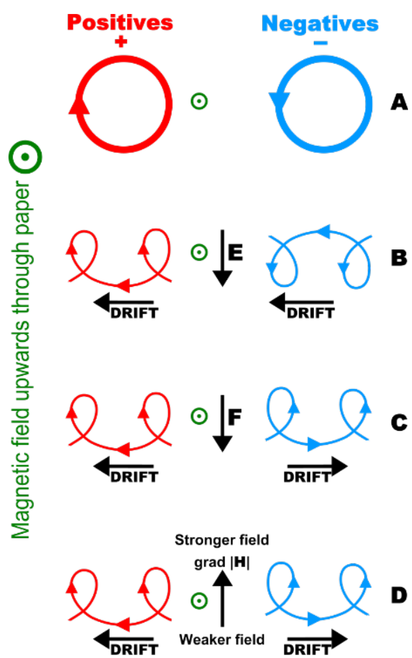


$$V_{\nabla B} = \pm \frac{1}{2} v_\perp r_L \frac{\vec{B} \times \nabla B}{B^2}$$

- **Curvature drift**

$$V_R = \frac{m v_{||}^2}{q} \frac{\vec{R}_c \times \vec{B}}{R_c^2 B^2}$$





$$V_R + V_{\nabla B} = \frac{m}{q} \left(v_{||}^2 + \frac{1}{2} v_\perp^2 \right) \frac{\vec{R}_c \times \vec{B}}{R_c^2 B^2}$$

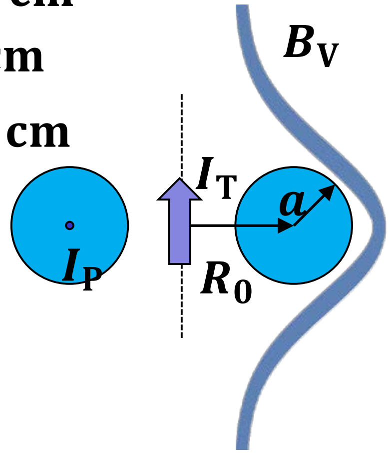
$$\approx \frac{1}{q} \left(2T_{||} + T_\perp \right) \frac{\vec{R}_c \times \vec{B}}{R_c^2 B^2}$$

https://en.wikipedia.org/wiki/Guiding_center
http://silas.psfc.mit.edu/introplasma/chap2.html

A vertical field B_V of 12 G with a curvature of 5 cm is needed to generate the required plasma current

- For P = 10^{-1} Torr = 13 Pa = 13 N/m^2

$$R_0 \sim 10\ \text{cm}$$
$$a \sim 5\ \text{cm}$$
$$R_C \sim 5\ \text{cm}$$



$$n = \frac{P}{T} = \frac{13}{4.1 \times 10^{-21}} = 3.1 \times 10^{21}\ m^{-3}$$

Assuming the ionization fraction is 1%:

$$n_e = n_i = 3.1 \times 10^{19}\ m^{-3}$$

$$j = qn_e v \qquad I \sim \pi a^2 j = \pi a^2 q n_e v$$

$$v = \frac{I}{\pi a^2 q n_e} = \frac{2 \times 10^3}{\pi 0.05^2 \times 1.6 \times 10^{-19} \times 3.1 \times 10^{19}} \sim 5 \times 10^4\ \text{cm/s}$$

$$v_{\text{drift}} = V_R + V_{\nabla B} \approx \frac{1}{q}\left(2T_{||} + T_{\perp}\right)\frac{\vec{R_c} \times \vec{B_V}}{{R_c}^2 {B_V}^2} \sim \frac{3T}{q}\frac{1}{R_C B_V}$$

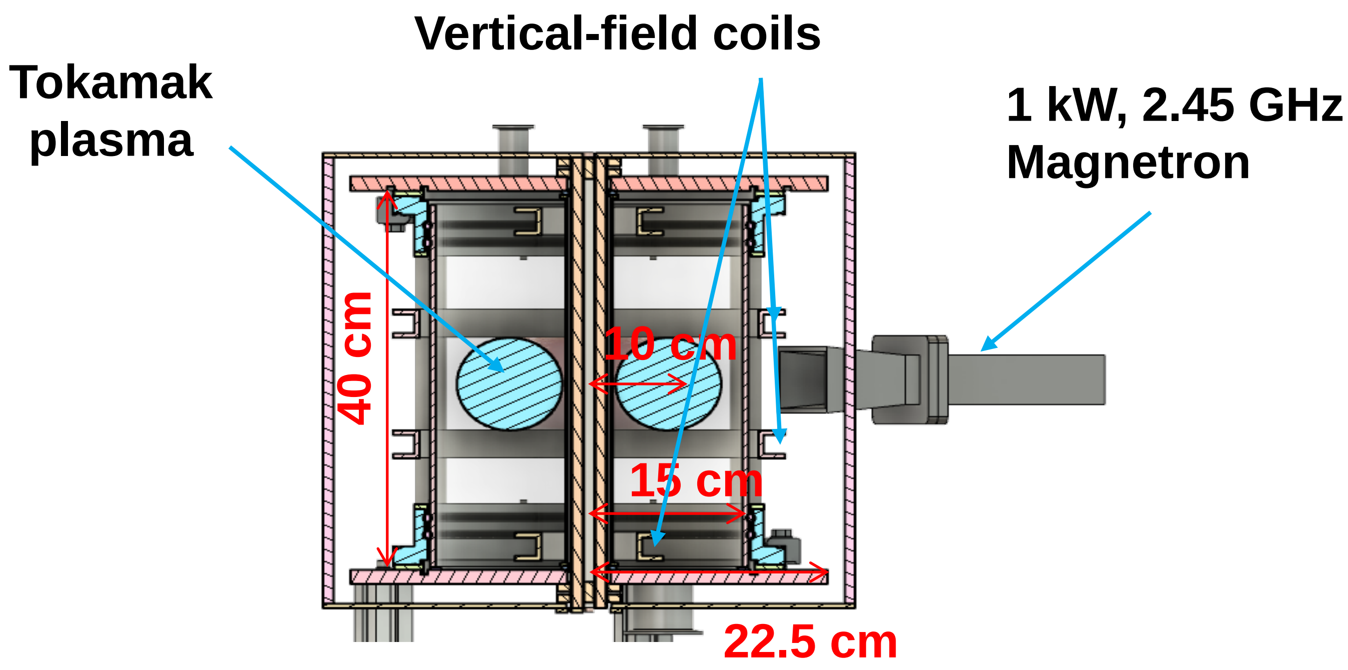
$$B_V \sim \frac{1}{V_{\text{drift}}}\frac{3T}{q}\frac{1}{R_C} \sim \frac{1}{5 \times 10^4} 3 \times 1 \times \frac{1}{0.05} \sim 0.0012\ \text{T} = 12\ \text{G} \quad (B_{\text{earth}} \sim 0.5\ \text{G})$$

- For T_e = 1 eV, B_V = 12 G

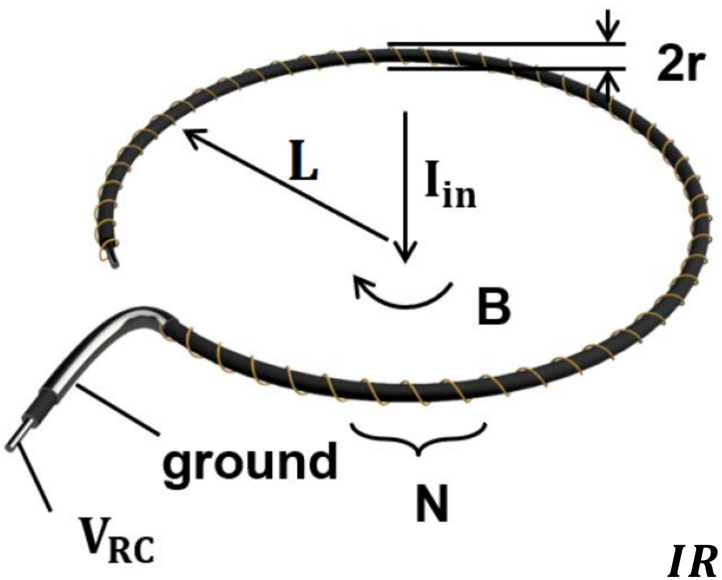
$$r_c = \frac{\sqrt{2\text{mT}}}{\text{e}B_V} = \frac{\sqrt{2 \times 9.11 \times 10^{-31} \times 1.6 \times 10^{-19}}}{1.6 \times 10^{-19} \times 0.0012} = 2.8\ \text{mm}$$

The prospective system design

(1) Vertical field coil (VF coil): B_V=12 G w/ curvature of 5 cm.
(2) Pulse forming network for driving VF coil: ? kA.
(3) Rogowski coil for measuring plasma current: I_P = 2 kA.
(4) Triple probe for measuring Plasma characteristics: Te~1 eV, $n_e \sim 10^{19}$ m^{-3}.



Current measurement using Rogowski coil



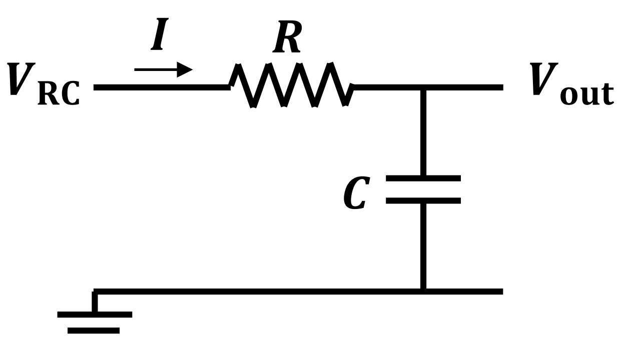
$$B = \frac{\mu_0 I_{\text{in}}}{2\pi L} \qquad \phi_1 \approx B \times \pi r^2 = \frac{\mu_0 r^2}{2L} I_{\text{in}}$$

$$\phi_{\text{Total}} = N \times \phi_1 = \frac{\mu_0 N r^2}{2L} I_{\text{in}}$$

$$V_{\text{RC}} = -\frac{d\phi_{\text{Toral}}}{\text{dt}} = -\frac{\mu_0 N r^2}{2L} \frac{dI_{\text{in}}}{dt}$$

$$I_{\text{in}} = -\frac{2L}{\mu_0 N r^2} \int_0^t V_{\text{RC}} \, \text{d}t'$$

$$IR + V_{\text{out}} = V_{\text{RC}} \qquad \frac{dV_{\text{out}}}{\text{dt}} + \frac{V_{\text{out}}}{\text{RC}} = \frac{V_{\text{RC}}}{\text{RC}} \qquad I = C \frac{dV_{\text{out}}}{\text{dt}}$$

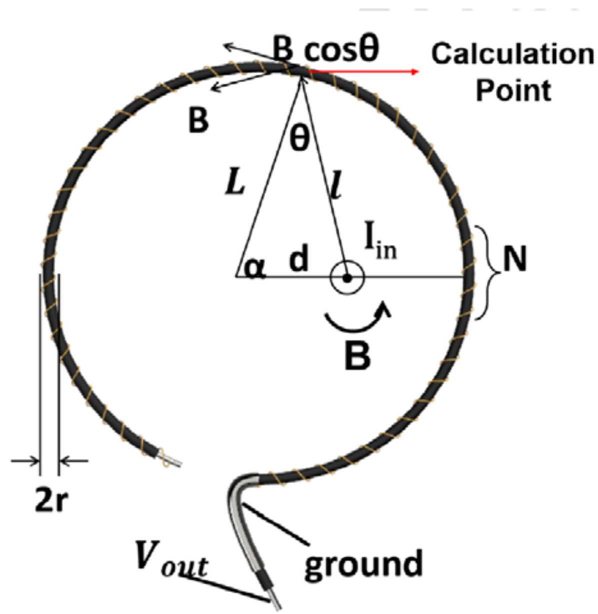


$$\frac{d\left(V_{\text{out}} e^{t/\text{RC}}\right)}{\text{dt}} = \frac{1}{\text{RC}} V_{\text{RC}} e^{t/\text{RC}}$$

$$V_{\text{out}} = \frac{e^{-t/\text{RC}}}{RC} \int_0^t V_{\text{RC}} e^{t/\text{RC}} \, \text{d}t \approx \frac{1}{RC} \int_0^t V_{\text{RC}} \, \text{d}t$$

$$I_{\text{in}} = -\frac{2L}{\mu_0 N r^2} \int_0^t V_{\text{RC}} \, \text{d}t' = -\frac{2LRC}{\mu_0 N r^2} V_{\text{out}}$$

The flux through the Rogowski coil is a weak function of the position of the current



$$B_{\text{eff}} = B\cos\theta = \frac{\mu_0 I_{\text{in}} \cos\theta}{2\pi l}$$

$$A = N\pi r^2$$

$$\frac{N\pi r^2}{2\pi L} = \frac{\Delta A}{\Delta \alpha} \equiv \frac{dA}{d\alpha} \quad \text{where } \alpha = 0 \text{ to } 2\pi$$

$$\phi_{\text{Total}} = \int_0^{2\pi} B_{\text{eff}} \frac{dA}{d\alpha}\, \mathrm{d}\alpha$$

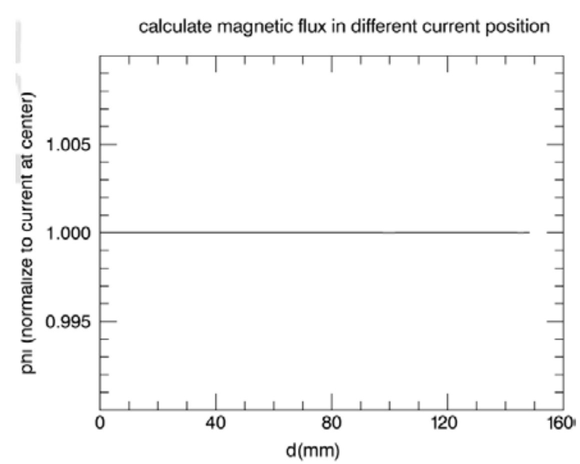
$$\phi_{\text{Total}} = \int_0^{2\pi} \frac{\mu_0 I_{\text{in}} \cos\theta}{2\pi l} \frac{N\pi r^2}{2\pi L}\, \mathrm{d}\alpha$$

$$l^2 = d^2 + L^2 - 2dL\cos\alpha$$

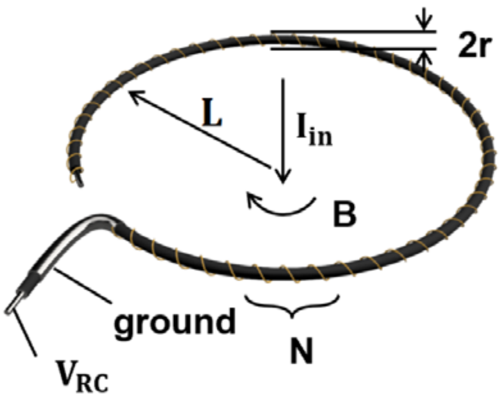
$$d^2 = l^2 + L^2 - 2lL\cos\theta$$

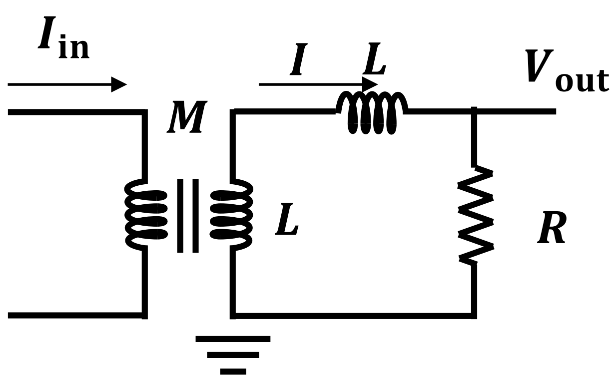
$$\cos\theta = \frac{l^2 - d^2 + dL\cos\alpha}{lL}$$

$$\phi_{\text{Total}}(d) = \int_0^{2\pi} \frac{\mu_0 I_{\text{in}} \left(L^2 - dL\cos\alpha\right)}{2\pi L(d^2 + L^2 - 2dL\cos\alpha)} \pi r^2 \frac{N}{2\pi L}\, \mathrm{d}\alpha$$



The Rogowski coil is self-integrated if the self inductance of the Rogowski coil is very large





$$V_{\mathrm{RC}} = -\frac{d\phi_{\mathrm{Toral}}}{\mathrm{dt}} = -\frac{\mu_0 N r^2}{2L}\frac{dI_{\mathrm{in}}}{dt} \equiv M\frac{dI_{\mathrm{in}}}{dt}$$

$$M\frac{dI_{\mathrm{in}}}{dt} - L\frac{dI}{dt} - IR = 0$$

$$V_{\mathrm{out}} = IR$$

$$\frac{dI}{dt} + \frac{R}{L}I = M\frac{dI_{\mathrm{in}}}{dt}$$

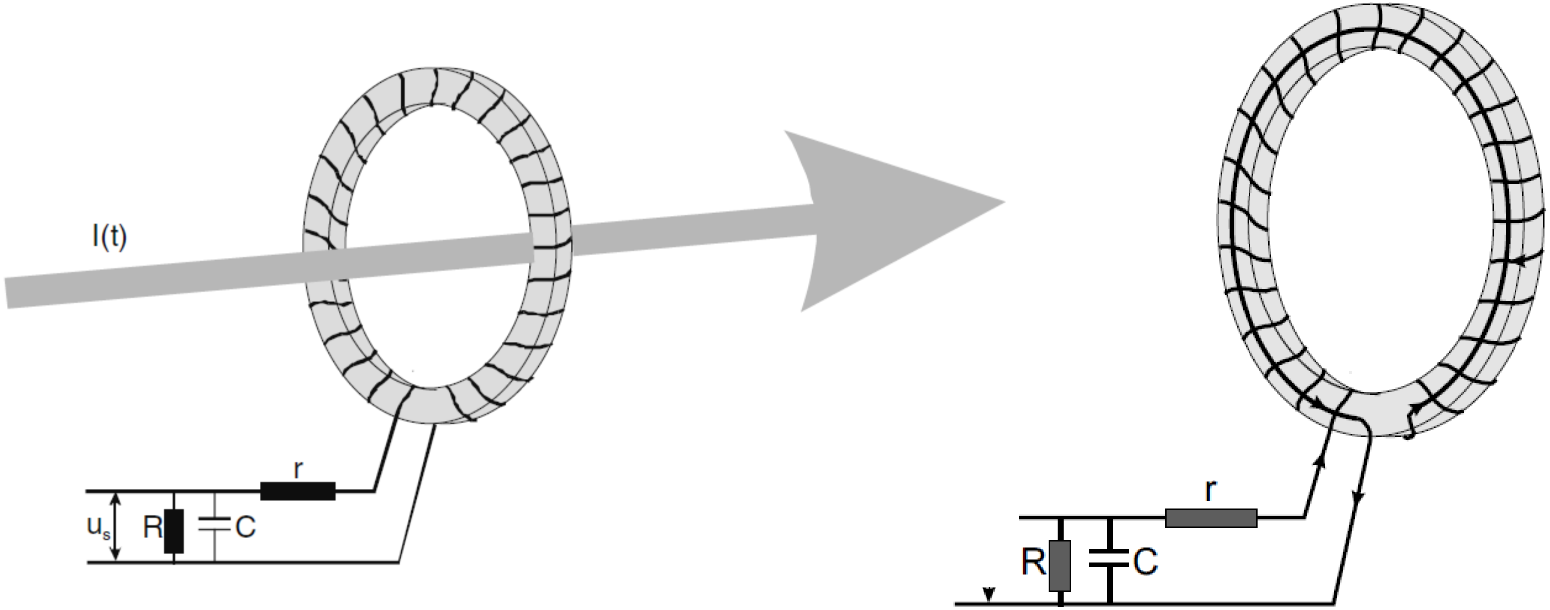
$$I + \frac{R}{\mathrm{i\omega L}}I = MI_{\mathrm{in}}$$

$$\text{For } \frac{R}{\omega L} << 1, \qquad I \approx MI_{\mathrm{in}}$$

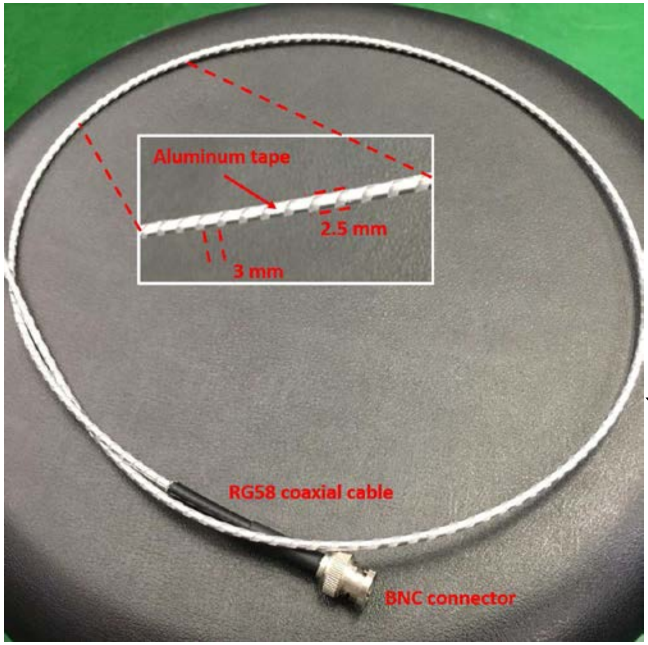
$$V_{\mathrm{out}} = RMI_{\mathrm{in}} = -\frac{\mu_0 N r^2}{2L}RI_{\mathrm{in}}$$

- **No integrated is needed!**

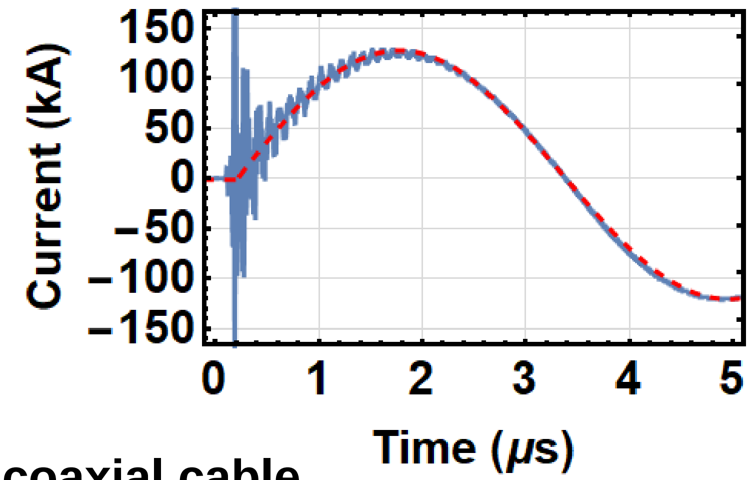
A big loop along the big radius of the Rogowski coil is needed to cancelled the flux contributed by the current that doesn’t pass through the coil

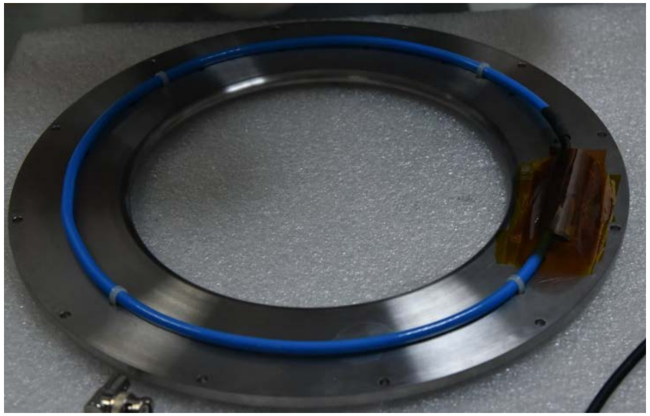


The Rogowski coil we built can measure current to more than 100 kA of current



RG58 coaxial cable





$$I_{\text{in}} = -\frac{2LRC}{\mu_0 N r^2} V_{\text{out}}$$

$f \sim 150\text{kHz}$

$I_{\text{peak}} \sim 100\text{kA}$

$L = 185\text{mm}$

$r = 1.25\text{mm}$

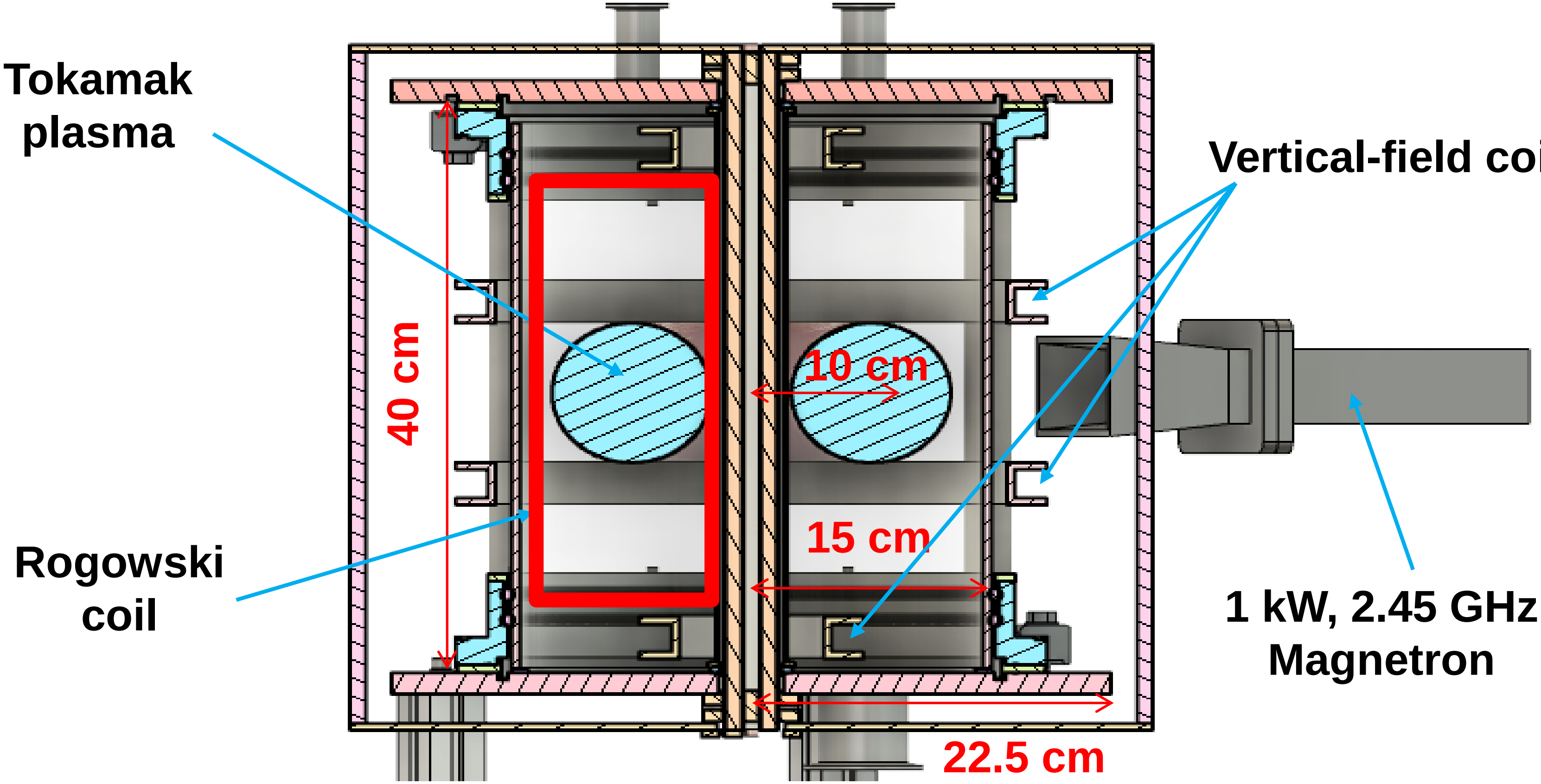
$N = 130$

$R = 5\ \text{k}\Omega$

$C = 48\ \text{nF}$

The Rogowski coil needs to wrap around the plasma

(1) Rogowski coil for measuring plasma current: I_P = 2 kA.



Class schedule

Week	Progress Description
1	2/25 簡介、分組、課程執行介紹
2	3/ 4 慣性控制核融合
3	3/11 磁場控制核融合
4	3/18 真空系統
5	3/25 電漿源
6	4/ 1 校慶(放假)
7	4/ 8 電漿加熱技術
8	4/ 15 脈衝功率系統
9	4/ 22 電漿量測

Week	Progress Description
10	4/ 29 小組討論
11	5/ 6 各組口頭報告設計
12	5/ 13 托克馬克各次系統實作
13	5/ 20 托克馬克各次系統實作
14	5/ 27 各組口頭報告進度
15	6/ 3 托克馬克各次系統實作
16	6/10 托克馬克各次系統實作
17	6/17 托克馬克實作
18	6/24 各組口頭報告實驗成果